lead to

$$\int_0^\pi \frac{d\theta}{(7+\cos\theta)^2} = \frac{7\sqrt{3}}{576}\pi$$

so that

$$V = \frac{80\pi}{3}(8 - 3\sqrt{3}) \text{ cu. in.}$$

$$V \doteq 234.894,585,349,6 \text{ cu. in.}$$

Also solved by G. A. KNAPP, C. N. SCHMALL, GEORGE PAASWELL, H. N. CARLETON, O. S. ADAMS, and the PROPOSER.

408. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

The ellipse $(x^2/81) + (y^2/16) = 1$ is revolved around the y-axis. Find the area of the surface generated.

SOLUTION BY CLYDE S. ATCHISON, Washington and Jefferson College.

From the given equation, we have

$$x = \frac{9}{4}\sqrt{16-y^2}; \quad dx = -\frac{9y\,dy}{4\sqrt{16-y^2}}; \quad \text{and} \quad ds = \frac{1}{4}\sqrt{\frac{256+65y^2}{16-y^2}}\cdot dy.$$

Then the area of the surface generated is

$$2\pi\int_{y=-4}^{y=4} x\cdot ds = 2\pi\int_{-4}^{4}\left(\frac{9}{4}\sqrt{16-y^2}\right)\left(\frac{1}{4}\sqrt{\frac{256+65y^2}{16-y^2}}\right)dy = \frac{9\pi}{8}\int_{-4}^{4}\sqrt{256+65y^2}\cdot dy$$

$$= \frac{9\pi\sqrt{65}}{16}\left[y\cdot\sqrt{y^2+\frac{256}{65}} + \frac{256}{65}\log\left(y+\sqrt{y^2+\frac{256}{65}}\right)\right]_{-4}^{4}$$

$$= \frac{9\pi\sqrt{65}}{16}\left\{\frac{144}{\sqrt{65}} + \frac{256}{65}\log\left(4+\frac{36}{\sqrt{65}}\right) + \frac{144}{\sqrt{65}} - \frac{256}{65}\log\left(-4+\frac{36}{\sqrt{65}}\right)\right\},$$

$$= 162\pi + \frac{144\pi}{\sqrt{65}}\log\frac{9+\sqrt{65}}{9-\sqrt{65}}.$$

Also solved by A. M. HARDING, NELLIE L. INGALS, HORACE OLSON, C. C. YEN, G. W. HARTWELL, H. C. FEEMSTER, J. A. ECKSON, GEORGE PAASWELL, O. S. ADAMS, and PAUL CAPRON.

MECHANICS.

321. Proposed by E. J. MOULTON, Northwestern University.

The attraction, A, in any direction, due to a homogeneous sphere, on a particle at the center of the sphere, using the Newtonian law, is obviously zero. Find the error in the following method of computing A. Take cylindrical coördinates with origin at the center of the sphere; let the Z-axis extend in the direction of the attraction to be computed, and let r, θ be the polar coördinates used. Let δ be the density and R the radius of the sphere, and k the constant of gravitation. Then

$$A = \int_{z=-R}^{z=R}\int_{r=0}^{r=\sqrt{R^2-z^2}}\int_{\theta=0}^{\theta=2\pi}\frac{k\delta r\,dz\,d\theta\,dr}{[r^2+z^2]^{\frac{3}{2}}} \tag{1}$$

$$= 2\pi k\delta\int_{z=-R}^{z=R}\left[\frac{-z}{(r^2+z^2)^{\frac{1}{2}}}\right]_{r=0}^{z=\sqrt{R^2-z^2}} dz \tag{2}$$

$$= 2\pi k\delta\int_{-R}^{R}\left[\frac{-z}{R}+1\right]dz \tag{3}$$

$$= 4\pi k\delta R. \tag{4}$$

SOLUTION BY ELIJAH SWIFT, University of Vermont.

In the integrand of (1) the radical is to be taken with the plus sign, as $[r^2 + z^2]^{\frac{1}{2}}$ is a length, namely, the radius vector from the origin to the element in question. In evaluating (2), then, we must take the radical with the plus sign. But when r is zero the value of the radical is $\pm z$, so that we must take $+ z$ when z is positive, $- z$ when z is negative. In obtaining (3), however, this value was taken as $+ z$ for all values of z. (3) should read

$$2\pi k\delta \left[\int_0^R \left[\frac{-z}{R} + 1 \right] dz + \int_{-R}^0 \left[\frac{-z}{R} - 1 \right] dz \right] = 0.$$

Also solved by PAUL CAPRON, J. W. CLAWSON, GEORGE PAASWELL, K. P. WILLIAMS, and the PROPOSER.

323. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Two equal bodies are placed on a rough inclined plane, being connected by a light inelastic string; if the coefficients of friction are respectively 1/3 and 1/4, show that they will both be on the point of motion when the inclination of the plane is $\sin^{-1}(7/25)$.

SOLUTION BY H. S. UHLER, Yale University.

In order to obtain the given inclination it is necessary to assume that the string lies in a vertical plane which is perpendicular to the intersection of the inclined plane with any horizontal plane, that it is taut, etc. From simple mechanical considerations it is evident that the body associated with the greater coefficient of friction must be higher up on the incline than its tandem body. Let m and T denote, respectively, the mass of either body and the tension of the string. By resolving the weight (mg) of each body parallel and perpendicular to the incline, and by employing the definition of the coefficient of friction, we find at once that the condition for being on the point of motion is expressed by the equations

$$T + mg \sin\alpha - \tfrac{1}{3}mg\cos\alpha = 0,$$

$$-T + mg\sin\alpha - \tfrac{1}{4}mg\cos\alpha = 0,$$

where α symbolizes the angle of elevation of the inclined plane. By first adding these equations and then dividing through by mg we get

$$2\sin\alpha - (7/12)\cos\alpha = 0,$$

or

$$\alpha = \tan^{-1}(7/24) = \sin^{-1}(7/25).$$

Also solved by J. ROSENBAUM, HORACE OLSON, H. C. FEEMSTER, C. A. NICKLE, G. PAASWELL, W. C. EELLS, J. A. CAPARO, W. H. THOME, and A. G. RAU.

NUMBER THEORY.

211. (April, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd perfect number exists, the total number of its divisors is a multiple of 2 but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$ where p is a prime and a is odd.

REMARK BY TRACY A. PIERCE, Harvard University.

Lucas in his *Théorie des Nombres* proved that an odd perfect number must be of the form $(4m + 1)^{4k+1}n^2$, where $4m + 1$ is a prime. See an article by BOURLET in *Nouv. Ann. de Math.*, 1896, pp. 297–312.

208. (March, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd number be perfect it cannot be the sum of two squares.